Fermi National Accelerator Laboratory

FERMILAB-Pub-82/73-THY
September 1982

Inverted Hierarchy and Asymptotic Freedom in Grand Unified Supersymmetric Theories*

HENRIK ARATYN†
Physics Department
Technion-Israel Institute of Technology
Haifa 32000, Israel

MOSHE MOSHE††
Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

ABSTRACT

The interrelation between an inverted hierarchy mechanism and asymptotic freedom in supersymmetric theories is analyzed in two models for which we performed a detailed analysis of the effective potentials and effective couplings. We find it difficult to accommodate an inverted hierarchy together with asymptotic freedom for the Matter-Yukawa couplings.

---

* Supported in part by the Israel Academy of Sciences and Humanities - Basic Research Foundation

† Supported by the Lady Davis Foundation

†† On leave (Aug. 1982 - Aug. 1983) from Physics Dept. Technion, Israel Inst. of Tech. Haifa, Israel.

One of the most puzzling and long lasting problems in theoretical high energy physics is the gauge hierarchy problem [1] [2]. Supersymmetry [3] provides a possible solution to this problem. This symmetry is, of course, broken at ordinary energies as dictated by the known particle spectrum. An interesting mechanism suggested by Witten[4], uses a variant of O'Raifeartaigh model [5] to generate at the tree level the necessary spontaneous breaking of supersymmetry at a scale $M < 10^{15}$ Gev. ($M \sim 10^{2}$ Gev in ref. 4 or 10^{12} Gev in the geometric hierarchy of ref. 6.) The large scale $\langle X \rangle$, presumably $\langle x \rangle \sim O(10^{15}$ Gev) is generated dynamically and thus provides the large mass needed in the theory.

The existence of Witten's inverted hierarchy scenario depends on the behavior of the coupling constants in the theory at large mass scale. Though asymptotic freeedom in all coupling constants is not a necessary condition for building a phenomenologically acceptable model, this property certainly helps either by justifying a low order calculation or neglecting the effect of certain reasonably small and decreasing couplings at various stages of the analysis. Thus, it is interesting to find out if the coupling constants involved in this type of models will not only produce the inverted hierarchy but also are (possibly all of them) asymptotically free. We present here a detailed analysis of two spontaneously broken supersymmetric models, their one loop effective potential and the behavior of their effective coupling constants at large scale. We discuss the consequences of demanding an inverted hierarchy combined with asymptotic freedom. The potential in supersymmetric theories is generated from a superpotential $W(A)$ in the form:

$$V(A) = \sum_i \left|\frac{\partial W}{\partial A_i}\right|^2 + \frac{1}{2}\sum_\alpha (D^\alpha)^2 , \tag{1}$$

where

$$D^\alpha = e_\alpha A^\dagger T^\alpha A \tag{2}$$

is the gauge interaction contribution, T^α is the generator of the group G in the given representation and e_α the associated gauge coupling. The necessary condition for the renormalizability of the theory is that W(A) should be a polynomial at most of order three

$$W(A) = a_i A^i + b_{ij} A^i A^j + c_{ijk} A^i A^j A^k$$

The absence of a solution to $\frac{\partial W}{\partial A_i} = 0$ for all i implies a spontaneous breakdown of supersymmetry at the tree level. The most quoted simple example is the O'Raifeartaigh model:

$$W(A, X, Y) = g\, X\,(A^2 - M^2) + \lambda\, Y A \quad (4)$$

in which $\frac{\partial W}{\partial X} = 0$ and $\frac{\partial W}{\partial Y} = 0$ cannot be satisfied simultaneously. The interesting feature in this model is that the vacuum expectation value (vev) of X, the scalar member of the superfield, is undetermined at the tree level. Radiative corrections through the one loop effective potential can fix the value of X and set a scale in the theory which could be much higher than the intrinsic scale M. Clearly, such a scenario is reminiscent of what is expected from a solution of the hierarchy problem[4]. Recently, as further steps towards building a realistic model, the following SU(N) extensions of Eq. 4 were widely employed:

$$W_1(A, X, Y) = g\, X\,(T_2 A^2 - M^2) - \lambda\, T_2 A^2 Y \quad (5)$$

$$W_2(A, X, Y) = g\, X\,(T_2 A^2 - M^2) - \lambda\, T_2 A Y \quad (6)$$

A and Y are in the adjoint representation and X is a singlet. Both have been studied first by Witten[7], for the SU(5) gauge group. The tree level potential $V_1(A, X, Y)$ derived from $W_1(A, X, Y)$ has a minimum at

$$A = gM(\lambda^2 + 30g^2)^{-\frac{1}{2}} \, \mathrm{diag}(2,2,2,-3,-3)$$

The vev's of X and Y are related to each other at the minimum of the potential by $Y = -(gX/\lambda)\mathrm{diag}(2,2,2,-3,-3)$ but the vev of X is undetermined at the tree level. The minimum for the potential V_2 of the second model occurs for $\mathrm{Tr}\,A^2 = (2g^2M^2 - \lambda^2)/2g^2$ with $Y = 2gXA/\lambda$ and the SU(5) breaking is not determined uniquely at this level[8].

The one-loop contribution to the effective potential is given by[10]:

$$\Delta V(\phi) = \sum_i \frac{(-1)^F}{64\pi^2} M_i^4(\phi) \ln\left(M_i^2(\phi)/\mu^2\right) \qquad (7)$$

where the sum is over all helicity states, M_i are the boson and fermion mass eigenvalue, μ^2 is a renormalization scale, F = 0 for bosons and F = 1 for fermions. The effective potential V_1 at large X is[4]:

$$V_1(X) = \frac{M^4 g^2 \lambda^2}{30g^2 + \lambda^2}\left[1 + \frac{g^2}{g^2 + \lambda^2/30}\,\frac{29\lambda^2 - 50e^2}{80\pi^2} \ln\left(|X|^2/\mu^2\right)\right] \qquad (8)$$

Indeed, if initially $H^2 \equiv \lambda^2/e^2 < 50/29$, the effective potential decreases as X increases until finally perturbation theory breaks down. However, if the Matter-Yukawa coupling λ becomes dominant with increasing X, the effective H^2 becomes larger than 50/29 and the coefficient of the logarithm in (8) will change sign. Thus necessarily a stable minimum will be created near the value X_0 for which $H^2(x_0) = 50/29$. If this picture is correct, a hierarchy of scales has been constructed with the large scale being generated dynamically at $X \sim M \exp(1/e^2)$, while the fundamental scale M is the supersymmetry breaking scale and can in principle be $M << X$. Clearly, as mentioned, the realization of this possibility depends on the behavior of the relevant coupling constants in the ultraviolet limit, which will be discussed later. We will first calculate the effective potential for the model in Eq. 6

In order to calculate the boson mass matrix it is convenient to decompose A as

$$A \longrightarrow A_0 + \frac{1}{\sqrt{2}}(A + iB)$$

with A displayed as 5x5 matrix $A = \sum_{i=1}^{24} A^i \lambda^i / \sqrt{2}$ with 8(8,1) fields, 3(1,3), one(1,1), 6(3,2) and 6($\bar{3}$,2) fields, where the entries (m,n) stand for the representations under the SU(3) and SU(2) subgroups. As in Witten's model[4] the contribution of the twelve (3,2) and ($\bar{3}$,2) fields to the mass matrix from the D-term is negative. The fermion mass matrix is determined by the Yukawa coupling $(\partial^2 W/\partial\phi_i \partial\phi_j)\psi^i\psi^j$ and the coupling between "gauginos" λ (the supersymmetric partners of Yang Mills fields) in the adjoint representations and the fermionic partners of A and Y fields. The mass matrices which we have to diagonalize, in order to evaluate the one-loop contribution in Eq. (7) are not larger than 3x3 matrices. A typical fermion mass matrix looks like

$$\mathcal{M}_F = \begin{pmatrix} 2gX & \lambda & \sqrt{30}\,g\,v \\ \lambda & 0 & 0 \\ \sqrt{30}\,g\,v & 0 & 0 \end{pmatrix} \tag{9}$$

where

$$v^2 = (2g^2M^2 - \lambda^2)/60\,g^2$$

The two left-handed and the two right-handed fermions associated with $\mathcal{M}_F$ have masses

$$\begin{aligned} m_\pm^2 &= 2g^2|X|^2 + \lambda^2 + 30g^2v^2 \pm g|X|\sqrt{4g^2|X|^2 + 4\lambda^2 + 120g^2v^2} \\ &\simeq 2g^2|X|^2 + \lambda^2 + 30g^2v^2 \pm 2g^2|X|^2 + O(\tfrac{1}{X}) \end{aligned} \tag{10}$$

One general constraint on the diagonal elements of $(\text{mass})^2$ matrices in the supersymmetric theories is given by the FGP mass relation [11], which in the absence of an extra U(1) gauge is given by

$$\mathrm{Tr}\,[m_B^2] + 3\,\mathrm{Tr}\,[m_V^2] = 2\,\mathrm{Tr}\,[m_F\, m_F^\dagger] \tag{11}$$

The FGP mass formula provided an extra check on our calculations. Substituting the masses of 60 real scalars, 12 vector bosons and 48 fermions, which depend on $|X|^2$, in Eq.(7) we find that the effective potential including the lowest order result is given by [12]:

$$V_2(x) = \lambda^2\left(M^2 - \frac{\lambda^2}{4g^2}\right) + \frac{\lambda^2}{16\pi^2}\left[-3\lambda^2 - 10\left(M^2 - \frac{\lambda^2}{2g^2}\right)e^2\right]\ln\frac{|X|^2}{\mu^2} \tag{12}$$

The X field will increase if the logarithm in (12) has a negative coefficient K:

$$K \equiv \frac{\lambda^4}{16\pi^2}\left[-3 - 10\,\frac{M^2 e^2}{\lambda^2} + 5\,\frac{e^2}{g^2}\right] \tag{13}$$

In order to have the tree level potential (derived from Eq. (6)) $V_2(\min)$ at $\mathrm{Tr}\,A^2 = (2g^2M^2 - \lambda^2)/2g^2$, $Y = 2gXA/\lambda$ smaller than V_2(min) at A=Y=0, we impose $\lambda/gM \ll 1$; this same condition results in $K<0$. Whether this coefficient will turn positive at lage X will depend here, as in Eq.

(8), on the detailed evolution of the effective λ, e, g and M at large X. This, along with the question of asymptotic freedom of the coupling constants for the two models in Eq. (5) and (6), will be discussed now.

In the case of the superpotential in Eq. (5) the effective coupling constants are determined [13] by:

$$\mu \frac{de}{d\mu} = \frac{e^3}{(4\pi)^2}\left[-(3-n)N + \frac{m}{2}\right] \tag{14a}$$

$$\mu \frac{d\lambda}{d\mu} = \frac{\lambda}{(4\pi)^2}\left[-6\,C_2(G)\,e^2 + 8g^2 + 5\,\frac{N^2-4}{N}\lambda^2\right] \tag{14b}$$

$$\mu \frac{dg}{d\mu} = \frac{g}{(4\pi)^2}\left[-4\,C_2(G)\,e^2 + (6+2N^2)g^2 + 4\,\frac{N^2-4}{N}\lambda^2\right] \tag{14c}$$

for the gauge group SU(N) with n and m the number of chiral superfields in adjoint and fundamental representation and $C_2(G) = N$.

In case of W_2 in Eq. (6) one obtains the following relations (Eq. 14a is not altered)

$$\mu \frac{d\lambda}{d\mu} = \frac{\lambda}{(4\pi)^2}\left(-4\,C_2(G)\,e^2 + 4g^2\right) \tag{15a}$$

$$\mu \frac{dg}{d\mu} = \frac{g}{(4\pi)^2}\left[-4\,C_2(G)\,e^2 + (6+2N^2)g^2\right] \tag{15b}$$

We will need also the evolution of M^2 in Eq. 12 . This is given by

$$\mu \frac{dM}{d\mu} = \frac{M}{(4\pi)^2} \left(C_2(G)\, e^2 - 2 g^2 \right) \tag{15c}$$

We will continue now the analysis for the SU(5) case. Equations 14(a),(b),(c) can be rewritten in the form:

$$\frac{dG^2}{d\tau} = \frac{G^2}{(4\pi)^2} \left(-32 + 112\, G^2 + 32.5\, H^2 \right) \tag{16a}$$

$$\frac{dH^2}{d\tau} = \frac{H^2}{(4\pi)^2} \left(-52 + 16\, G^2 + 42\, H^2 \right) \tag{16b}$$

where $G^2 = g^2/e^2$, $H^2 = \lambda^2/e^2$ and τ is the scale parameter,

$$\tau = \int_0^{t = \ln\mu} dt'\, e^2(t')$$

Clearly, if initially we choose $-52 + 16G^2 + 42H^2 > 0$ then H^2, as well as G^2 increases as $\tau \to \infty$. The increase of H^2 is necessary for Witten's scenario to work in Eq. (8). If H^2 and G^2 increase, then either λ and g are increasing or e^2 decreases faster than these two coupling constants[f1]. Looking now back to Eqs. 14(b)(c), we see that the later possibility would imply that asymptotically both $\mu \frac{d\lambda}{d\mu}$ and $\mu \frac{dg}{d\mu}$ are positive and thus λ and g are non-asymptotically free. Writing an equation for $\mu \frac{dR}{d\mu}$ where $R = \lambda/g$ shows that R decreases provided that initially

$\lambda \leq g$. This last condition assures also that the correct tree level minimum of $V_1(A)$ has been chosen. Note that the inclusion of higher orders in the renormalization group equations could in principle alter this picture. However, as long as the involved couplings are small (e.g. α_{GUT}^{-1} for supersymmetric GUT is about 24) we can use the renormalization group equations in one-loop approximation to study the evolution of the coupling constants at larger scales[f2].

For the superpotential W_2 in Eq. (6) we have the set of equations (14a), (15a-c). In general, if

$$\mu \frac{de}{d\mu} = b_0 e^3 \tag{16a}$$

$$\mu \frac{dg}{d\mu} = A g^3 + B e^2 g \tag{16b}$$

with $A>0$ and $B<0$, then clearly for $B-b_0<0$ there are two fixed points: one at $g^2/e^2 = G^2 = -(B-b_0)/A$ and the other, the ultra-violet stable one, at $G^2=0$. Thus, for $g^2 < [(b_0-B)/A]\,e^2$, the Matter-Yukawa coupling g^2 goes to zero more rapidly than e^2 as $\mu \to \infty$. Thus we have also $\lambda \to 0$ and $M \to \infty$, as $\mu \to \infty$ as seen in Eqs. (15a) and (15c). Asymptotic freedom for g and λ in this model is feasible due to the fact that there is "less" matter interaction in W_2 than in W_1. For example, there is no contribution in the lowest order to the anomalous dimension of the superfield Y from interaction with other superfields; the anomalous dimension of the A superfield is also reduced here.

In order to see whether the asymptotically free coupling constants in this case can also produce Witten's inverted hierarchy, Eq. (13) has to be examined. The differential equation for $\tilde{m}^2 \equiv M^2 e^2/\lambda^2$ is

$$\mu \frac{d\tilde{m}^2}{d\mu} = 2 \frac{\tilde{m}^2}{(4\pi)^2} \left\{ \left[6\, C_2(G) - (3-n) N + \frac{m}{2} \right] e^2 - 8 g^2 \right\} \tag{17}$$

and thus $\tilde{m}^2 \rightarrow \infty$ as $\mu \rightarrow \infty$. Now, to check $\tilde{K} \equiv -3 - 10\tilde{m}^2 + 5 G^{-2}$ in Eq. (13), we note also that $\mu \frac{d\tilde{R}}{d\mu} > 0$, where $\tilde{R} = \tilde{m}^2 G^2$. Thus, if the coefficient K of $\ln |X|$ is initially negative, then we see that also asymptotically it stays negative and Witten's scenario is not guaranteed in this case. Note, however, that the reversal of the sign of K at an asymptotic scale is not a necessary condition for an inverted hierarchy scenario since it is enough that nearby $\langle X \rangle \sim O(10^{15}\ \mathrm{GeV})$ $K(x)$ becomes positive and later at larger scales it approaches its asymptotic negative regime. Such behavior of the expression in Eq. (13), though it is obtainable in principle (by tunning the coupling constants and their initial condition properly), may be at least as difficult to implement in a realistic model as is the necessity to work with non-asymptotically free coupling in the case of W_1 .

We were interested above merely in the existence of an inverted gauge hierarchy, namely, whether one can prove that indeed the coefficient of $\ln x$ necessarily changes sign. This, of course, is only a necessary condition to produce Witten's hierarchy, the precise value that $\langle x \rangle$ obtains, the representation one has to use etc.(14) , are very important problems and difficulties one has to overcome in building realistic models.

In conclusion, the analysis of two superpotentials presented above shows an interesting interrelation between two properties: (a) inverted hierarchy, and (b) asymptotic freedom of the Matter-Yukawa couplings. We found that it is difficult to accommodate one together with the other. The superpotential W_1 in Eq. (5) will necessarily produce an inverted hierarchy scenario. We proved, however, that all Yukawa couplings are non- asymptotically free. On the other hand, in W_2 of Eq. (6) where all Yukawa couplings are asymptotically free, we have proved, however, that Witten's scenario may be implemented only by tunning parameters; it does not necessarily exist as in the case of W_1 . From the experience with the above calculations it seems to us that this may very well be a general feature of supersymmetric grand unifying theories. Whether one can show that properties (a) and (b) above are mutually excluded (or are at least hardly implementable together) in any SUSY GUT is left here as an interesting theoretical open question.

Upon completion of our work, we received several papers which deal with some of the aspects discussed here as well as related topics[15].

ACKNOWLEDGEMENTS

We wish to acknowledge stimulating and fruitful discussions with many of our colleagues.

REFERENCES

[1]E. Gildener and S. Weinberg, Phys. Rev. D13 (1976) 3333. E. Gildener, Phys. Rev. D14 (1976) 1667.

[2]L. Susskind, Phys. Rev. D20 (1979) 2619.

[3]For a review of Supersymmetry see P. Fayet and S. Ferrara, Phys. Reports 32C (1977) 249. J. Wess and J. Bagger, Supersymmetry and Supergravity, Lectures given at Princeton University, Spring 1981.

[4]E. Witten, Phys. Let. 105B (1981) 267.

[5]L. O'Raifeartaigh, Nucl. Phys. B96 (1975) 331.

[6]S. Dimopoulos and S. Raby, Los Alamos preprint (1982).

[7]E. Witten, Nucl. Phys. B188 (1981) 513.

[8]Adding two Higgs multiplets to W_2 $fH_{\bar{5}}AH_5 + mH_{\bar{5}}H_5$ still leaves several symmetry breaking patterns feasible. If $\langle H_5\rangle_5 \neq 0$ and we require $m/f=3v$, where $v^2 = (2g^2M^2-\lambda^2)/60g^2$ (such an adjustment will be protected by supersymmetric non-renormalization theorems of ref.9), then the constraints on the possible symmetry breaking pattern are

$$\sum_{i=1}^{4} c_i = 3v \quad , \quad \sum_{i=1}^{4} c_i^2 = 21 v^2$$

where $A = \mathrm{diag}(C_1 C_2 \ldots C_5)$. Thus the $SU(5) \to SU(3) \times SU(2) \times U(1)$ is only one of several possible solutions.

[9]J. Wess and B. Zumino, Phys. Lett. 49B (1974) 52. J. Iliopoulos and B. Zumino, Nucl. Phys. B 76 (1974) 310. S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77

(1974) 413. M.T. Grisaru, W. Siegel and M. Rocek, Nucl. Phys. B159 (1979) 429.

[10] S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888.

[11] S. Ferrara, L. Girardello and F. Palumbo, Phys. Rev. D20 (1979) 403.

[12] The SO(3) variant of this model was investigated by P. Ginsparg, Phys. Lett. 112b (1982) 45.

[13] D.R.T. Jones, Nucl. Phys. B87 (1975) 127. K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Kyushu preprint 82-HE-2R (1982).

[14] H. Georgi, Phys. Lett. 108B (1982) 283.

[15] T. Banks and V. Kaplunovsky, Tel Aviv preprint TAUP 1028-82. M.B. Einhorn and D.R.T. Jones, Michigan preprint UM HE 82-19. H. Yamagishi, Princeton University preprint.

FOOTNOTES

Footnote 1 (f1): Problems due to the presence of light color octet fields were widely discussed recently. See for example, refs. 14,15.

Footnote 2 (f2): This issue has been studied by Banks and Kaplunovsky (ref. 15).